PARTICLE SWARM DESIGN OPTIMIZATION OF MOMENT RESISTING
STEEL FRAMES WITH SEMI-RIGID CONNECTIONS TO LRFD-AISC

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1. Abstract
Particle swarm optimization algorithm is proven to be reliable and efficient technique for obtaining the solution of discrete structural optimization problems. In this study, particle swarm algorithm is used to develop an optimum design method for semi-rigid steel frames. The design algorithm presented selects W-sections for the members of steel frame from the complete list of W-sections given in LRFD-AISC (Load and Resistance Factor Design, American Institute of Steel Construction). The design constraints are implemented from the specifications of the same code which covers serviceability and strength limitations. The selection of W-sections is carried out such that the design limitations are satisfied and the weight of semi-rigid frame is the minimum. Two steel frames designed by the optimum design algorithm presented to demonstrate its efficiency. 

Keywords: optimum design, minimum weight, steel frame, combinatorial optimization, swarm intelligence, particle swarm optimizer.

2. Introduction
In the analysis and design of steel frames, the realistic modeling of beam-to-column connections provides an accurate response of the frame under the external loads. In practice, these connections are assumed to be either fully rigid or perfectly pinned. In the former assumption, it is implied that there is no relative rotation of connection and the column takes the whole end moment of the beam. On the other hand, the pinned connection assumes that the moment of connection is always zero and there is no existing restraint for rotation of the connection. However, experiments have revealed that the real behavior of beam-to-column connections is between these extremes. Namely, all these practically used connections possess some stiffness falling between two cases mentioned above. Moreover, it is found that there exists a nonlinear relation of relative beam-to-column rotation when a moment is applied to a flexible connection [1]. These partially restrained connections influence the drift (P-Δ effect) of whole structure as well as the moment distribution in beams and columns. Use of a direct nonlinear inelastic analysis is one way to account for all these effects in frame design [6]. To be able to implement such analysis, beam-to-column connections should be assumed and modeled as semi-rigid connections. The semi-rigid connection flexibility depends on the geometric parameters of the elements used in beam-to-column connection such as dimensions of end plates and bolt size.

The present study develops a particle swarm optimizer based optimum design method for unbraced steel frames with semi rigid connections. Particle swarm method based optimum design of steel frames with rigid end connections is carried out in [14]. The method is one of the recent additions to meta-heuristic techniques that are based on swarm intelligence. It is population based optimization algorithm. Its population is called a swarm and each individual in the swarm is called a particle. Each particle flies through the problem space to search for optimum. Hence number of particles which are initialized randomly in the search space of an objective function is initially considered in particle swarm optimizer. Each particle represents a potential solution of the optimization problem and it has two attributes. One is its current position in the search space and the other is its current velocity. Position and velocity of each particle are updated at a time period to reach the best position in the search space.

Optimum design problem is formulated according to the specifications of LRFD-AISC [15]. The design constraints; displacement limitations, inter-storey drift restrictions of multi-storey frames, strength requirements for beam-columns are included in the formulation of the design problem. Furthermore, additional constraints namely geometric constraints are also considered to satisfy the practical requirements. The design problem formulated turns out to be a discrete nonlinear programming problem. The design algorithm developed selects W-sections for beams and columns of an unbraced frame from discrete steel section list such that the design constraints imposed by LRFD-AISC are satisfied and the frame has the minimum weight.

3. Particle Swarm Optimizer for Discrete Design variables
The optimum design problem with discrete variables can be expressed as follows:
Min. \( f(x_i) \) \( i = 1, \ldots, n \)

Subject to:\[
g_j(x_i) \leq 0 \quad j = 1, \ldots, m
\]
\[
x_i \in X, \quad X = \{x_1, x_2, \ldots, x_q\}
\]

where \( x_i \) represents the discrete design variable which is to be selected from the set \( D \) that contains \( q \) number of discrete values for these variables. \( n \) is the total number of design variables. \( f(x_i) \) is the objective function and \( g_j(x_i) \) is the design constraint \( j \), \( m \) is the total number of these constraints in the design problem.

Particle swarm optimization is a population based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling. The procedure involves a number of particles which represents the swarm are initialized randomly in the search space of an objective function. Each particle in the swarm represents a candidate solution of the optimum design problem. Originally particle swarm optimizer is developed for continuous design variables. To be able to use the method for discrete design variables some adjustments are required to be carried out. Firstly the discrete values among which the values of design variables \( x_i \) are to be selected in set \( X \) are arranged in ascending sequence. The sequence number of these values is then treated as design variables instead of \( x_i \) itself. At any stage of design cycle, once a sequence number is generated by the algorithm, then the real value of the design variable which corresponds to this sequence number is easily taken from the discrete set. The particle swarm optimizer for discrete variables consists of six basic steps.

**Step 1.** Swarm of particles is initialized randomly with sequence numbers \( I_0 \) that correspond to positions \( x_0 \) and initial velocities \( v_0 \) that are randomly distributed throughout the design space. Here \( I_0 \) represents the sequence number of values in the discrete set. These are obtained from the following expressions.

\[
I_0^i = INT[I_{\text{min}} + r(I_{\text{max}} - I_{\text{min}})] \quad (2)
\]
\[
v_0^i = [(I_{\text{min}} + r(I_{\text{max}} - I_{\text{min}}))]/\Delta t) \quad (3)
\]

where the term \( r \) represents a random number between 0 and 1, \( I_{\text{min}} \) is equal to 1 and \( I_{\text{max}} \) is the total number of values in the discrete set respectively. Once \( I_0^i \) is computed, the corresponding \( x_0^i \) value is taken from the discrete set.

**Step 2.** The objective function values \( f(x_0^i) \) are evaluated using the design space positions \( x_0^i \).

**Step 3.** The optimum particle position \( p_k^i \) at the current iteration \( k \) and the global optimum particle position \( p_k^g \) are updated by equating \( p_k^i \) to \( f(x_k^i) \) and \( p_k^g \) to the best \( f(x_k^i) \).

**Step 4.** The velocity vector of each particle is updated considering the particle’s current velocity and position, the particle’s best position and global best position, as follows:

\[
v_{k+1}^i = wv_k^i + c_1r_1\left(\frac{p_k^i - x_k^i}{\Delta t}\right) + c_2r_2\left(\frac{p_k^g - x_k^i}{\Delta t}\right) \quad (4)
\]

where \( r_1 \) and \( r_2 \) are random numbers between 0 and 1, \( p_k^i \) is the best position found by particle \( i \) so far, and \( p_k^g \) is the best position in the swarm at time \( k \). \( w \) is the inertia of the particle which controls the exploration properties of the algorithm. \( c_1 \) and \( c_2 \) are trust parameters that indicate how much confidence the particle has in itself and in the swarm respectively.

**Step 5.** The sequence number for the position of each particle is updated from
\[ I'_{k+1} = \text{INT} \left( I'_i + v'_{k+1} \Delta t \right) \]  

Where \( I'_{k+1} \) is the sequence number in the discrete set for \( x'_{k+1} \), which is the position of particle \( i \) at iteration \( k+1 \), \( v'_{k+1} \) is the corresponding velocity vector and \( \Delta t \) is the time step value.

**Step 6.** Steps 2-5 are repeated until the pre-determined maximum number of cycles is reached.

**Constraint handling:** In this study fly-back mechanism is used for handling the design constraints which is proven to be effective in [16]. Once all particle positions \( x \) are generated, the objective functions are evaluated for each of these and the constraints in the problem are then computed with these values to find out whether they violate the design constraints. If one or a number of the particle gives infeasible solutions, these are discarded and new ones are re-generated. If some particles are slightly infeasible then such particles are kept in the solution. These particles having one or more constraints slightly infeasible are utilized in the design process that might provide a new particle that may be feasible. This is achieved by using larger error values initially for the acceptability of the new design vectors and then reduce this value gradually during the design cycles and uses finally an error value of 0.001 or whatever necessary value that is required to be selected for the permissible error term towards the end of iterations. This adaptive error strategy is found quite effective in handling the design constraints in large design problems.

### 4. Discrete optimum design of unbraced steel frames with semi-rigid connections to LRFD-AISC

In the design of unbraced steel frames, the main concept is to select the readymade steel sections for its columns and beams from standard steel section tables. This design can be valid only if the serviceability and strength requirements specified by the code of practice are satisfied. In order to obtain an accurate response of the frame under the external loading, beam-to-column connections are assumed to be partially restrained. Hence, the stability analysis is included in the formulation of the design problem.

When the constraints are implemented from LRFD–AISC in the formulation of the design problem the discrete optimum design problem of unbraced steel frames with semi-rigid beam-to-column connections of which the objective is the minimum weight can be expressed as follows:

Minimize

\[ W = \sum_{k=1}^{ng} m_k \sum_{i=1}^{nk} L_i \]  

Subject to

\[ (\delta_j - \delta_{j-1})/h_j \leq \delta_{ju} \quad j = 1, \ldots, ns \]  

\[ \delta_i \leq \delta_{iu} \quad i = 1, \ldots, nd \]  

\[ V_{il} \leq \phi V_n \]  

\[ \left( \frac{P_{il}}{\phi c P_n} \right) + \left( \frac{8 (M_{il})}{\phi b M_{nx}} \right) \leq 1.0 \quad \text{for } \frac{P_{il}}{\phi c P_n} \geq 0.2 \]  

\[ \left( \frac{P_{il}}{2\phi c P_n} \right) + \left( \frac{M_{il}}{\phi b M_{nx}} \right) \leq 1.0 \quad \text{for } \frac{P_{il}}{\phi c P_n} \leq 0.2 \]  

\[ B_{sb} \leq B_{sc} \quad s = 1, \ldots, nu \]  

\[ D_s \leq D_{s-1} - 1 \]  

\[ m_s \leq m_{s-1} \]

where Eq. (6a) defines the weight of the frame, \( ng \) is total numbers of groups in the structural system, \( m_s \) is the unit weight of the steel section selected from the standard steel sections table that is to be adopted for group
k, L is the length of member i that belongs to group k, nk is total number of members in group k. Eq. (6b) represents the inter-storey drift of the multi-storey frame. \( \delta_j \) and \( \delta_{j-1} \) are lateral deflections of two adjacent storey levels and \( h_i \) is the storey height. ns is the total number of storeys in the frame. Eq. (6c) defines the displacement restrictions that may be required to include other than drift constraints such as deflections in beams. nd is the total number of restricted displacements in the frame. \( \delta_{ia} \) is the allowable lateral displacement. The horizontal deflection of columns is limited due to unfactored imposed load and wind loads to height of column / 300 in each storey of a building with more than one storey. \( \delta_{ia} \) is the upper bound on the deflection of beams which is given as span / 300 if they carry plaster or other brittle finish. Eq. (6d) represents the shear capacity check for beam-columns. \( \phi \) is resistance factor in shear, \( V_s \) required shear strength, \( V_n \) is nominal shear strength. Eq. (6e) defines the local capacity check for beam-columns. \( n_m \) is number of members, \( n_l \) is number of load cases, \( M_{ns} \) is nominal flexural strength, \( M_{ns} \) is applied moment, \( P_n \) is nominal axial strength, \( P_a \) is applied axial load, \( \Theta_k \) is resistance factor for columns if the axial force is in compression, \( \Theta_b \) is resistance factor in bending. It is apparent that computation of compressive strength \( \phi P_n \) of a compression member requires its effective length. The computation of the effective length of a compression member in a frame can be automated by using Jackson and Moreland monograph [17]. Eq. (6f) is included in the design problem to ensure that the flange width of the beam section at each beam-column connection of storey \( s \) should be less than or equal to the flange width of column section. Eqs. (6g) and (6h) are required to be included to make sure that the depth and the mass per meter of column section at storey \( s \) at each beam-column connection are less than or equal to width and mass of the column section at the lower storey \( s-1 \). \( m_u \) is the total number of these constraints.

5. Analysis of unbraced steel frames with semi-rigid connections
There exist various semi-rigid connection modeling and their moment-rotation relationships in the literature. The most well-known ones are linear, polynomial, cubic B spline, power and exponential models. In the analysis and design of semi-rigid steel frames connections can be represented by discrete, inelastic rotational springs. The effect of connection flexibility is modeled by attaching rotational springs with stiffness moduli \( K_A \) and \( K_B \) to the first and second ends of a member as shown in Figure 1.

Figure 1. Semi -rigid plane beam member with rotational springs. (a) End forces and end displacements (b) end rotations.
A beam member with semi-rigid end connections has the nonlinear stiffness matrix form shown in the following.

\[
[S] = \begin{bmatrix}
A & B & C1 & E1 & F1 \\
B & D & -A & -B & -C1 \\
-C1 & -B & A & D & E1 \\
-C2 & -E2 & F2 & C2 & E2 & G
\end{bmatrix}
(7)
\]

Where

\[
A = \frac{EA}{L} + \frac{12EI}{L^2} f_{s1} \phi_3 \\
B = \frac{EA}{L} - \frac{12EI}{L^2} f_{s1} \phi_3 \\
D = \frac{EA}{L} - \frac{12EI}{L^2} f_{s1} \phi_3 \\
C_1 = \frac{6EI}{L^2} f_{s2} \phi_2 \\
C_2 = \frac{6EI}{L^2} f_{s3} \phi_2 \\
E_1 = \frac{6EI}{L^2} f_{s5} \phi_2 \\
E_2 = \frac{4EI}{L} f_{s6} \phi_3 \\
E_3 = \frac{2EI}{L} f_{s5} \phi_3
\]
in which E represents the modulus of elasticity, L, I, A are the length, moment of inertia and area of beam respectively. Above stiffness matrix includes the effect of the flexible connections. To be able to modify the stiffness matrix of rigid beam modification coefficients are used. These coefficients are calculated using following equations.

In Eqn. (9) $K_d$ and $K_b$ symbolize the stiffness moduli of the flexible connections at first and second end of the member. In addition stability functions are included in the stiffness matrix to consider the effect of axial forces on the deformed shape. To calculate the values of stability functions power series approximation is used. However, this method needs the trigonometric functions and one of which is $\alpha \cot \alpha$ gives singular values at some $\alpha$ values. For this reason Livesely’s approximation [18] which is the sum of a power series in Euler critical load factor $\rho$ and a rotational function Eqn. (10) is implemented. These stability functions are given as follows;

$$\phi_i = \alpha \cot \alpha = \frac{64-60\rho+5\rho^2}{(16-\rho)(4-\rho)} - \sum_{i=1}^{7} a_i \rho^i$$

(10)

where $P$ is the axial force in beam member, $P_{cr}$ is the Euler critical load of beam member.

In practice, curve-fitting the experimental data with simple expressions is the most commonly used approach to describe the $M$-$\theta$ curve of flexible connection. There are several analytical models to represent connection flexibility using available experimental test data. A polynomial model where $M$-$\theta$ behavior is represented by an odd power polynomial, proposed by Frye and Morris [1], is adopted in present study due to its easy implementation. The Frye and Morris model uses the method of least square to determine the constants of the polynomial and has the following form. (Eqn.12)

$$\theta_t = C_1(KM)^1 + C_2(KM)^3 + C_3(KM)^5$$

(12)

in which $C_1$, $C_2$, $C_3$ are the curve-fitting constants and $K$ symbolizes the standardization constant dependent on connection type and geometry. The values of these constants vary for each connection type and are given in [19]. $K_d$ and $K_b$ called as the rotational stiffness of the springs at each end of the flexible frame member are calculated as a tangent stiffness using above given nonlinear standardized function. (Eqn. 12) To achieve this, first flexibility of connection is determined as $d\theta_t/dM$. Then, the stiffness of the connection which is to be used in the modification of general stiffness matrix is obtained as a reciprocal of the connection flexibility calculated for a certain value of a moment, when connection is loaded [19]. If the state is unloading, the stiffness of the connection is assumed as its initial stiffness. These two states are shown in Figure 2. The size parameters of the end plate without column stiffeners connection modeled in this study are shown schematically in Figure 3. For the end plate without column stiffeners connection model, curve-fitting and standardization constants are given as in the following. (Eqn. 13)

$$C_1 = 1.83 \times 10^{-3} \quad C_2 = 1.04 \times 10^{-4} \quad C_3 = 6.38 \times 10^{-6} \quad \text{and} \quad K = d_g^{-2.4} t_p^{-0.4} d_h^{-1.5}$$

(13)
where $d_g$, $t_p$, $d_b$ are; the distance between two bolts at the top and bottom of end plate, the thickness of plate and the diameter of bolts respectively.

An increase in lateral displacements occurs in the analysis of steel frames with semi-rigid connections. Hence, consideration of the effect of axial forces in the response of semi-rigid frame becomes a necessity. The following steps give details about the algorithm which accounts for P-Δ effect in the analysis of frame.

1. In the beginning of the procedure, axial forces in the frame members are assumed to be zero.
2. Overall stiffness matrix is constructed then the frame is analyzed under the external loads. Joint displacements and member forces are calculated.
3. Corresponding stability functions are determined using the axial forces obtained for the members. The steps from 2 are repeated until the difference between two successive sets of axial forces is smaller than a specific tolerance.

The determinant of overall stiffness matrix is calculated and the loss of stability is checked during these iterations. If no loss of stability occurs and the convergence in the axial forces is obtained, the joint displacements and member forces determined in this nonlinear response are used in the computation of fitness values for this particle. During the analysis the design load is applied immediately and the iterations are carried out at this load. It should be pointed out that fixed end moments change in each iteration due to the rotational springs. The modified fixed end moments are determined by considering the flexible end connection.

6. Design examples

Two unbraced semi-rigid steel frames are designed using particle swarm method based optimum design algorithm presented in the previous section. The discrete set from which the design algorithm selects the sectional designations for frame members is considered to be the complete set of 272 W-sections starting from W100x19.3 to W1100x499mm as given in LRFD-AISC.

6.1 Four storey- four bay steel frame

The four storey four bay steel frame shown in figure 4 is considered as first example. The frame consists of thirty six members that are collected in seven groups as shown in the figure. The allowable inter-storey drift is 1 cm while the lateral displacement of the top storey is limited to 4 cm. The modulus of elasticity is 200kN/mm².
The optimum W-sections designations obtained by the particle swarm method are given in Table 1. Frame is analyzed assuming the beam-to-column connections to be first end plate without column stiffeners then rigid. The optimum results obtained for designs with rigid and semi-rigid connections are presented in Table 1. The comparison of the minimum weight of frames for each case is shown in Fig. 5. Fig. 5 implies that frames with semi-rigid connections (end plate without column stiffeners) are heavier than ones with rigid connections. The design when connections are assumed to be semi rigid is determined after 270 iterations and the minimum weight of the frame is 6747.59 kg while it is 5890.52 kg in the case of rigid connection. It can be clearly seen from the results that maximum inter storey drifts are dominant in each design.
Table 1: Optimum designs for four-storey, four-bay steel frame

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Member Type</th>
<th>Frame analysis</th>
<th>Rigid</th>
<th>End plate without column stiffener</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Column</td>
<td>W200x52</td>
<td>W250X67</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Column</td>
<td>W200x46.1</td>
<td>W250X49.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Column</td>
<td>W310x52</td>
<td>W310X67</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Column</td>
<td>W200x46.1</td>
<td>W310X67</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Beam</td>
<td>W360x39</td>
<td>W360X57.8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Beam</td>
<td>W200x26.6</td>
<td>W200X26.6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Beam</td>
<td>W410x60</td>
<td>W360X44</td>
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<table>
<thead>
<tr>
<th></th>
<th>Maximum inter storey drift ratio</th>
<th>Maximum Strength Ratio</th>
<th>Top storey drift (cm)</th>
<th>Minimum Weight (kg)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.93</td>
<td>0.73</td>
<td>2.73</td>
<td>5890.52</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.93</td>
<td>3.05</td>
<td>6747.59</td>
</tr>
</tbody>
</table>

6.2 Five storey three bay steel frame

The five-storey, three-bay frame of Fig. 6 is also designed using the algorithm presented herein. The frame configuration dimensions, loading, joint numbering and member grouping is shown in the figure. This frame is designed with and without considering $P-\Delta$ effect together with rigid and semi-rigid connection modeling. Results are shown in table 2. Similar to first example, semi-rigid connection modeling gives heavier design. Allowable inter-storey drift is again 1 cm and the top storey sway was limited to 5 cm. The strength ratios obtained are 0.58 and 0.78 and top storey drifts are 3.64 and 2.96 for rigid and semi-rigid frames respectively. Maximum inter-storey drift ratios are 0.91 and 0.80 respectively. This clearly indicates that maximum inter-storey drift ratios dominate the designs. Design-history graph of both assumptions are shown in figure 7.

Figure 6: Four-storey, four-bay steel frame
Table 2: Optimum designs for five-storey, three-bay steel frame

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Member Type</th>
<th>Frame analysis</th>
<th>End plate without column stiffener</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Column</td>
<td>W360X32.9</td>
<td>W610X92</td>
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<tr>
<td>2</td>
<td>Column</td>
<td>W200X22.5</td>
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<td>W310X23.8</td>
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<td>6</td>
<td>Beam</td>
<td>W360X32.9</td>
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<td>7</td>
<td>Beam</td>
<td>W310X32.7</td>
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<table>
<thead>
<tr>
<th></th>
<th>Maximum inter storey drift ratio</th>
<th>Maximum Strength Ratio</th>
<th>Top storey drift (cm)</th>
<th>Minimum Weight (kg)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.91</td>
<td>0.58</td>
<td>3.64</td>
<td>1873.84</td>
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<td></td>
<td>0.80</td>
<td>0.78</td>
<td>2.96</td>
<td>2690.43</td>
</tr>
</tbody>
</table>

7. Conclusion
In this study, a particle swarm optimizer based optimum design algorithm is presented for unbraced steel frames with semi-rigid connections. Developed computer program analyzes the nonlinear steel frame with flexible connections taking into account the LRFD-AISC specifications and selects W sections from ready section lists available in the literature. Experimental studies in the literature have shown that connection flexibility affects the distribution of forces in the frame and leads to an increase in the drift of whole structure. This makes it necessary to consider P-Δ effect in the frame analysis. There are several connection models in the literature. Among them end plate without column stiffeners connection model is carried out in the present study. Results show that optimum semi-rigidly connected frames are heavier than rigidly connected ones.

8. Acknowledgement
This paper is partially based on research supported by the Scientific Research Council of Turkey (TUBITAK Research Grant No:108M070) which is gratefully acknowledged.
9. References